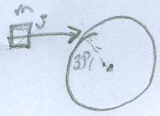
EVF 2011 - 2Sem 25/03/2016

15:00

Q1.

a) 

$\tau = I \cdot \alpha$ $\quad x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$a_c = \frac{v^2}{R}$

$\boxed{v_{cm} = \frac{\sum m_i v_i}{\sum m_i}} \Rightarrow (M+m) v_{cm} = mv$

$\boxed{v_{cm} = \frac{mv}{M+m}}$

b) $L_i = L_f$ just before hits

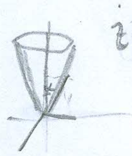
$mvR = I\omega_f \Leftrightarrow mvR = \omega_f\left(\frac{1}{2}MR^2 + mR^2\right)$

$I = \frac{1}{2}MR^2 + mR^2$ $\Downarrow$

$\omega_f = \frac{mv\cancel{R}}{\frac{1}{2}R^2(M+m)} = \frac{2v}{R}\left(\frac{1}{1+\frac{M}{m}}\right)$

c) $E_i = \frac{mv^2}{2}$?

$E_f = \frac{I}{2} \cdot \omega_f^2 \Rightarrow \Delta E = I\omega_f^2 - \frac{mv^2}{2} =$

Q2. a) 

$z = r \cdot \cot\alpha$ $\quad x = r\cos\theta$ $\quad y = r\sin\theta$

$\dot{z} = \cot\alpha \cdot \dot{r}$ $\quad \dot{x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta$ $\quad \dot{y} = \dot{r}\sin\theta + r \cdot \dot{\theta}\cos\theta$

$\dot{x}^2 = \dot{r}^2\cos^2\theta - 2\dot{r}r\dot{\theta}\cos\theta\sin\theta + r^2\dot{\theta}^2\sin^2\theta$

$\dot{y}^2 = \dot{r}^2\sin^2\theta + 2\dot{r}r\dot{\theta}\cos\theta\sin\theta + r^2\dot{\theta}^2\cos^2\theta$

$\Big\} \dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2\dot{\theta}^2$ $(1+\cot\alpha)$

$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{r}^2\cot^2\alpha) = \boxed{\frac{1}{2}m\dot{r}^2\csc^2\alpha + \frac{1}{2}mr^2\dot{\theta}^2}$

$\boxed{U = mgz = mgr\cot\alpha}$

b) $L = T - V = \frac{m\dot{r}^2}{2}\cdot \csc^2\alpha + \frac{1}{2}mr^2\dot{\theta}^2 - mgr\cot\alpha$

for r:

$m\ddot{r}\csc^2\alpha - mr\dot{\theta}^2 + mg\cot\alpha = 0$

$\ddot{r} - r\dot{\theta}^2\sin^2\alpha + g\cos\alpha\cdot\sin\alpha = 0$

for θ:

$mr^2\ddot{\theta} - 0 = 0 \Rightarrow mr^2\ddot{\theta} = 0 \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} = 0 \Rightarrow \frac{\partial L}{\partial\dot{\theta}} = mr^2\dot{\theta} = \text{constant}$

c) Since, θ cyclic, ignorable coordinate

$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} = 0 \Rightarrow \frac{\partial L}{\partial\dot{\theta}} = mr^2\dot{\theta} = \text{constant}$

$mr^2\omega = mrv = L$; maybe energy because $\frac{\partial L}{\partial t} = 0$

d) $H = \sum p_i\cdot\dot{q}_i - L$

$p_r = \frac{\partial L}{\partial\dot{r}} = m\dot{r}\csc^2\alpha \Rightarrow \dot{r} = \frac{p_r}{m}\sin^2\alpha$

$p_\theta = \frac{\partial L}{\partial\dot{\theta}} = mr^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{p_\theta}{mr^2}$

$H = (p_r\cdot\dot{r} + p_\theta\cdot\dot{\theta}) - \frac{m\dot{r}^2}{2}\csc^2\alpha - \frac{1}{2}mr^2\dot{\theta}^2 + mgr\cot\alpha$

$H = \frac{p_r^2\sin^2\alpha}{m} + \frac{p_\theta^2}{mr^2} - \frac{m p_r^2\sin^{\cancel{2}}\alpha}{m^2 2} - \frac{1}{2}\frac{mr^2\cdot p_\theta^2}{m^2 r^2{}^2} + mgr\cot\alpha$

$\boxed{= \frac{p_r^2\sin^2\alpha}{2m} + \frac{p_\theta^2}{2mr^2} + mgr\cot\alpha}$

$\dot{p}_r = \frac{\partial H}{\partial r} = \frac{p_\theta^2}{mr^3} - mg\cot\alpha \qquad \dot{\theta} = \frac{p_\theta}{mr^2}$

$\dot{p}_\theta = -\frac{\partial H}{\partial\theta} = 0 \qquad \dot{r} = \frac{\partial H}{\partial p_r} = \frac{p_r\sin^2\alpha}{m}$

$E_m = \frac{m\dot{r}^2}{2}\csc^2\alpha + \frac{1}{2}mr^2\dot{\theta}^2 + mgr\cot\alpha$

e) oscilações radiais

$V = \frac{1}{2}kr^2$ $V = mg r \cos\alpha$

$\dot{V} = kr$, $\ddot{V} = K$ $\dot{V} = mg \cos\alpha =$

$\ddot{V} = 0 = K$

$\boxed{f = \frac{1}{2\pi} \sqrt{\frac{3L^2}{m r_0^4}}}$?

ou

$V_{eff} = \frac{L^2}{2mr^2} + mg r \cos\alpha$

$\omega^2 = \frac{k}{m}$

$\dot{V} = -\frac{L^2}{mr^3} + mg \cos\alpha \Rightarrow \ddot{V} = -\frac{L^2}{r_0^3} \cdot mg\cos\alpha \Rightarrow \frac{3L^2}{m r_0^4} = K$

Q3.

a) r_0 e r_1 são os pontos de equilíbrio estável dos potenciais. $\left(\frac{dV}{dr} = 0\right)$

b) É esperado que a molécula B passe essa energia em segundos, emitindo um fóton e voltando p/ o estado fundamental [Inicialmente a molécula está oscilando em torno do ponto de equilíbrio. Quando ela muda p/ o 1º estado eletrônico a tendência é que a molécula pare de oscilar e aumente a sua distância (aumentando r) ...] $r_1 < r_2$

c) $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ 1D $\Rightarrow P = \int |\psi(x)| \, dx$; Volume 3D $= \int |\psi(r)|^2 dV$

radial probability distribution is:

$P(r) = \frac{1}{\pi a_0^3} \cdot e^{-\frac{2r}{a_0}} \cdot \overbrace{4\pi r^2 \cdot dr}^{\frac{4}{3}\pi(r+dr)^3 - \frac{4}{3}\pi r^3}$ $\frac{1}{2\pi a_0^2}$ $1 - \frac{r}{a_0} \Rightarrow \boxed{r = a_0}$

$\frac{dP}{dr} = 0 = 8\pi r \, e^{-2r/a_0} - \frac{8\pi r^2}{a_0} e^{-2r/a_0}$

d) $\langle r \rangle = \int_0^\infty \psi(r)^* \cdot r\, \psi(r)\, dr = \frac{4\cancel{\pi}}{\cancel{\pi} a_0^3} \int_0^\infty r^3 \cdot e^{-\frac{2r}{a_0}}\, dr$

$u = r^3 \quad dv = e^{-\frac{2r}{a_0}}$

$du = 3r^2 dr \quad v = -\frac{a_0}{2} e^{-\frac{2r}{a_0}}$

I

$r^3 \cdot \left(-\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} + \int \frac{3a_0}{2} e^{-\frac{2r}{a_0}} \cdot r^2\, dr$ (II)

$u = r^2 \quad dv = e^{-\frac{2r}{a_0}}$

$du = 2r\,dr \quad v = -\frac{a_0}{2} e^{-\frac{2r}{a_0}}$

II

$r^2 \cdot \left(-\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} + \int \frac{+a_0}{\cancel{2}} e^{-\frac{2r}{a_0}} \cdot \cancel{2}r\, dr$

$u = r \quad dv = e^{-\frac{2r}{a_0}}$

$du = dr \quad v = -\frac{a_0}{2} e^{-\frac{2r}{a_0}}$

III

$r \cdot \left(-\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} + \int \frac{+a_0}{2} e^{-\frac{2r}{a_0}} \cdot dr$

$= r\left(\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} - \frac{a_0^2}{2^2} e^{-\frac{2r}{a_0}}$

$$\langle r \rangle = \frac{4}{a_0^3} \left\{ \frac{-r^3 a_0}{2} e^{-\frac{2r}{a_0}} + \frac{3a_0}{2} \left[\frac{-a_0 r^2}{2} e^{-\frac{2r}{a_0}} + a_0 \left(\frac{-a_0 r}{2} e^{-\frac{2r}{a_0}} - \left(\frac{a_0}{2}\right)^2 e^{-\frac{2r}{a_0}} \right) \right] \right\}_0^\infty$$

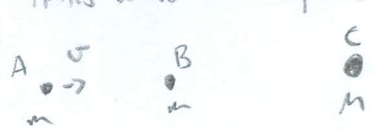
$$= \frac{4e^{-\frac{2r}{a_0}}}{a_0^3} \left[-\frac{r^3 a_0}{2} - \frac{3a_0^2 r^2}{4} - \frac{3a_0^3 r}{4} - \frac{3a_0^4}{8} \right]_0^\infty$$

$\boxed{\langle r \rangle = \frac{3a_0}{2}}$

0,69

Q4. a)

Antes colisão. Depois colisão



A. $p = \gamma m v$. $\gamma = \frac{5}{3}$

$= m \cdot \frac{4}{5} c \cdot \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{4}{\cancel{8}} mc \cdot \frac{\cancel{8}}{3} = \boxed{\frac{4}{3} mc}$ ✓ $\boxed{E_A = \frac{5}{3} mc^2}$ ✓

B.

$p = 0$; $\boxed{E_B = m_0 c^2}$ ✓

C.

$\boxed{E = E_A + E_B = \frac{8}{3} m_0 c^2}$ ✓

$\boxed{p = \frac{4}{3} mc}$

0,8

0,7 = 0,56

$\boxed{4}$

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b) $\frac{v_c}{c^2} = \frac{p_c}{E_c} \Rightarrow v_c = \frac{\frac{4}{3}mc^3}{\frac{4}{3}mc^2} = \boxed{\frac{c}{2}}$

$p_c = \gamma M v_c = \frac{4}{3}mc$

$E_c = \gamma M c^2 = \frac{4}{3}mc^2$

$\gamma_1 = \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{3}}$

c) $\frac{4}{3}mc = \gamma M v_c \Rightarrow \frac{4}{3}m\cancel{c} = \frac{2}{\sqrt{3}}M\frac{\cancel{c}}{2} \Rightarrow M = \frac{4\sqrt{3}}{3}m \Rightarrow \boxed{M = \frac{4m}{\sqrt{3}}}$

Q5.

a) $dU = dQ - dW \qquad p = \frac{nRT}{V}$

$ncdT = TdS - pdV$

$dS = \frac{nc\,dT}{T} - \frac{nR\,dV}{V} \Rightarrow \boxed{\Delta S = nc\ln\left(\frac{T_1}{T_2}\right) - nR\ln\left(\frac{V_1}{V_2}\right)}$

b) $A \to B$

$dU = dQ - dW \Rightarrow W = \Delta Q$

$\Delta U = 0$

$p_1V_1 - p_4V_4 = 0$

$p_1V_1 = p_4V_4$

$\frac{nR\Delta T}{V}$

$\boxed{W_1 = \int_{V_1}^{V_2} p\,dV = nRT_q \ln\left(\frac{V_2}{V_1}\right) = -Q_1}$

$C \to D$

$W_3 = nRT_f \ln\left(\frac{V_4}{V_3}\right) = -Q_3$

∂U

$Q = nc\Delta T + p\hat{V}$

$\cancel{\partial V}$

$Q = n\Delta T(c_v + R)$

$\boxed{Q = n\Delta T \cdot c_p}$

$B \to C \quad \Delta U = -W$

$\boxed{Q_2 = 0 \quad W_2 = nc\Delta T_1}$

$D \to A$

$Q_4 = 0 \quad W_4 = nc\Delta T_2$

$\Delta T_1 = T_f - T_q = -\Delta T_2$

c) $\eta = \frac{W_1}{Q_1} = \frac{Q_1 - Q_3}{Q_1} = 1 - \frac{Q_3}{Q_1} = \boxed{1 - \frac{T_f}{T_q}}$

↳ qtd de calor fornecido

pelo facto que é convertido p/ trabalho.

$\Delta U = 0 \Rightarrow W = \Delta Q = Q_1 - Q_3$

$$\frac{Q_1}{Q_3} = \frac{T_q}{T_f}$$

Q6.

a)

$r < R$

$$E = \frac{Q_{enc}}{2\pi r l \varepsilon_0} = \frac{r\rho}{2\varepsilon_0}$$

$Q_{enc} = \pi r^2 l \rho$

$r > R$

$$\boxed{E = \frac{R^2 \rho}{2 r \varepsilon_0}}$$

$Q_{enc} = \pi R^2 l \rho$

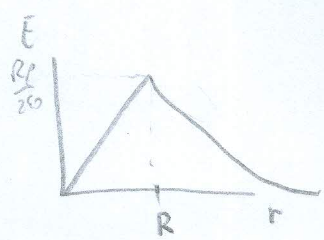
b) $r < R$

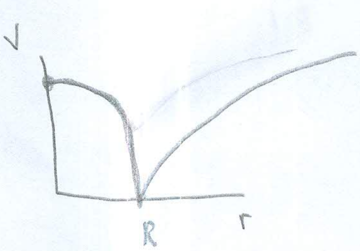
$$V = \int_R^r -E\,dl = \int_R^r -\frac{r\rho}{2\varepsilon_0}\,dr = -\frac{\rho}{2\varepsilon_0} \cdot \left(\frac{r^2 - R^2}{2}\right)$$

$r > R$

$$V = \int_R^r -\frac{R^2\rho}{2r\varepsilon_0}\,dr = \frac{R^2\rho}{2\varepsilon_0}\left[\ln\left(\frac{r}{R}\right)\right]$$

c)





(6)

d) Podemos aumentar a rigidez dielétrica do ar, ou seja, acrescentar um dielétrico na tubulação p/ aumentar $\underline{\underline{K}}$ (constante dielétrica)

$$\boxed{E = \frac{Q_{enc}}{2\pi r l K \cdot \varepsilon_0}}$$

Q7.

a) $\vec{E} = E_0 \cdot e^{i(\vec{k}\vec{r} - \omega t)}$

$$\boxed{\nabla \vec{E} = ik\vec{E}}$$

$$\boxed{\frac{\partial \vec{E}}{\partial t} = -i\omega \vec{E}}$$

$H = E_0' \cdot e^{i(k\vec{z} - \omega t)}\hat{y}$

$E = E_0 \cdot e^{i(k\vec{z} - \omega t)}\hat{x}$

$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

$$\begin{cases} ik \times \vec{E} - i\omega \vec{B} = 0 \\ ik \cdot B = 0 \\ ikD = \rho \\ ik \times H + i\omega D = J \end{cases}$$

$ik \times H + i\omega \varepsilon \vec{E} = J$

b) $\rho = n$

$\underline{ik \times H} + \frac{i}{k}\omega n = J$

$\frac{\langle e \rangle}{V} = n \quad \rho = \frac{Q}{V}$

$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ ik & ik & H \\ 0 & H & 0 \end{vmatrix} = ik\vec{H}\hat{z} - ikH\hat{x}$

$ik\vec{H}\hat{z} - ikH\hat{x} + \frac{i}{k}\omega n = J$

$J = \frac{i}{k}\omega n$

c)

Q8.

a) $\frac{\partial \rho}{\partial t} = -\frac{\partial j}{\partial x}$ $\qquad \frac{\partial \rho}{\partial t} = \frac{\partial[\psi^*(x,t)\cdot\psi(x,t)]}{\partial t} = \frac{\partial \psi^*(x,t)}{\partial t}\cdot\psi(x,t) + \psi^*(x,t)\cdot\frac{\partial \psi(x,t)}{\partial t}$

$$= -\frac{\partial j}{\partial x}$$

$$\boxed{i\hbar\cdot\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x,t)}$$

$$i\hbar\left(\frac{\partial \rho}{\partial t}\cdot\frac{1}{\psi^*(x,t)} - \frac{\partial \psi^*(x,t)}{\partial t}\cdot\frac{\psi(x,t)}{\psi^*(x,t)}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x,t)$$

using

$$\boxed{-i\hbar\frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + V\psi^*} \rightarrow \frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + \frac{iV\psi^*}{\hbar}$$

$$i\hbar\left(\frac{\partial \rho}{\partial t}\cdot\frac{1}{\psi^*} + \left(\frac{i\hbar}{2m}\cdot\frac{\partial^2 \psi^*}{\partial x^2} - \frac{iV\psi^*}{\hbar}\right)\cdot\frac{\psi}{\psi^*}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$$

$$i\hbar\frac{\partial \rho}{\partial t}\cdot\frac{1}{\psi^*} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \cancel{V(x)\psi(x)} + \frac{\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\cdot\frac{\psi}{\psi^*} - \cancel{V(x)\psi}$$

$$\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2}\cdot\psi^* + \cancel{\frac{i}{\hbar}V(x)\cdot\rho(x,t)} - \frac{i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\cdot\psi + \cancel{V(x)\cdot\rho(x,t)}$$

$$\boxed{\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m}\left[\frac{\partial^2 \psi}{\partial x^2}\cdot \psi^* - \frac{\partial^2 \psi^*}{\partial x^2}\cdot \psi\right]} + \frac{\partial \psi^*}{\partial x}\cdot\frac{\partial \psi}{\partial x} + \psi^*\cdot\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial x^2}\cdot\psi - \frac{\partial \psi^*}{\partial x}\cdot\frac{\partial \psi}{\partial x}$$

b) $$-\frac{\partial j}{\partial x} = \frac{i\hbar}{2m}\left[\frac{\partial^2 \psi}{\partial x^2}\cdot \psi^* - \frac{\partial^2 \psi^*}{\partial x^2}\cdot \psi\right]$$

$$-\frac{\partial j}{\partial x} = \frac{i\hbar}{2m}\frac{\partial}{\partial x}\left[\psi^*\frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x}\cdot\psi\right]$$

$$\boxed{j = -\frac{i\hbar}{2m}\cdot\left[\psi^*\frac{\partial \psi}{\partial x} - \frac{\partial \psi^*}{\partial x}\psi\right]}$$

c) $\langle x\rangle = \int \psi^*\cdot x\psi \, dx \qquad \langle p\rangle = \int \psi^*(-i\hbar)\cdot\frac{\partial \psi}{\partial x}dx$

$$\frac{d\langle x\rangle}{dt} = \int \frac{\partial(\psi^*\cdot x\psi)}{\partial t}dx = \int\left[\frac{\partial \psi^*}{\partial t}\cdot x\psi + \psi^*\cdot\frac{\partial x\psi}{\partial t}\right]dx$$

$$= \int\left\{\left[\frac{-i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + \cancel{\frac{i}{\hbar}V\psi^*}\right]\cdot x\psi + \psi^*\cdot x\left[\frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2} - \cancel{\frac{i}{\hbar}V\psi}\right]\right\}dx$$

$$= \int\left[\frac{-i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\cdot x\psi + \frac{i\hbar}{2m}\psi^*\cdot x\frac{\partial^2 \psi}{\partial x^2}\right]dx = -\frac{i\hbar}{2m}\int\left(x\cdot\psi\frac{\partial^2 \psi^*}{\partial x^2} - x\psi^*\frac{\partial^2 \psi}{\partial x^2}\right)dx$$

$$\frac{d\langle x\rangle}{dt} = \frac{+i\hbar}{2m}\int x\left[-\psi\cdot\frac{\partial^2\psi^*}{\partial x^2} + \psi^*\cdot\frac{\partial^2\psi}{\partial x}\right]dx \qquad \langle p\rangle = \int \psi^*\cdot(-i\hbar)\frac{\partial\psi}{\partial x}dx$$

$$= \frac{-i\hbar}{2m}\int x\cdot\frac{\partial}{\partial x}\left[\psi^*\cdot\frac{\partial\psi}{\partial x} - \psi\cdot\frac{\partial\psi^*}{\partial x}\right]dx$$

$u = x \qquad dv = \frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right]$

$du = dx$

$v = \psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}$

$$= \frac{-i\hbar}{2m}\left[\underbrace{\cancel{\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)\cdot x\Big|_{-\infty}^{\infty}}}_{0} - \int \psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\,dx\right]$$

$\left[\psi\frac{\partial\psi}{\partial x}\to\infty \text{ nu ajib' ger } \frac{1}{x}\right]$

$\left[\frac{1}{x}\frac{1}{x}\cdot x\right] = \frac{1}{x}$

$$\Rightarrow \frac{i\hbar}{2m}\int_{-\infty}^{\infty}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)dx$$

$$= \frac{1}{2m}\left(\langle p\rangle + \int_{-\infty}^{\infty} i\hbar\psi\frac{\partial\psi^*}{\partial x}dx\right) = \frac{1}{2m}\left(\langle p\rangle + \langle p\rangle\right) = \boxed{\frac{\langle p\rangle}{m}}$$

$u = \psi \quad dv = \frac{\partial\psi^*}{\partial x}i\hbar$

$du = \frac{\partial\psi}{\partial x} \quad v = i\hbar\psi^*$

$$= \overset{0}{\cancel{i\hbar\psi^*\cdot\psi}} - \underbrace{\int_{-\infty}^{\infty} i\hbar\psi^*\frac{\partial\psi}{\partial x}dx}_{} = \langle p\rangle$$

Q9.

a) $$\hat{N}\hat{a}|n\rangle = \hat{a}^\dagger\hat{a}\cdot\hat{a}|n\rangle = \left[\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\right]\hat{a}|n\rangle$$

$$= \left[[\hat{a}^\dagger,\hat{a}] + \hat{a}\hat{a}^\dagger\right]\hat{a}|n\rangle = -\hat{a}|n\rangle + \hat{a}n|n\rangle = (n-1)\hat{a}|n\rangle$$

autoestado!

autovalor $\Rightarrow n-1 = n'$

$$\hat{N}\hat{a}^\dagger|n\rangle = \hat{a}^\dagger\cdot\hat{a}\cdot\hat{a}^\dagger|n\rangle = \hat{a}^\dagger\cdot\left[\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\right]|n\rangle$$

$$= \hat{a}^\dagger\left[1 + \hat{a}^\dagger\hat{a}\right]|n\rangle = \underline{(n+1)\hat{a}^\dagger|n\rangle}, \quad \text{autovalor} \Rightarrow n+1 = n''$$

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b) $\boxed{\langle n|n\rangle = 1}$ $\boxed{\hat{a}|n\rangle = c_n|n'\rangle}$

$n = \langle n|N|n\rangle = \langle n|\hat{a}^\dagger\hat{a}|n\rangle = \langle \hat{a}n|\hat{a}n\rangle$

$\langle n|\hat{a}^\dagger\hat{a}|n\rangle = c_n\langle n|\hat{a}^\dagger|n-1\rangle$

$\langle n|\hat{a}|n\rangle = n\cdot 1$

$c_n\langle n-1|\hat{a}|n\rangle^* = |c_n|^2$

$\langle n-1|n-1\rangle = 1$

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$

$n > 0$

$|c|^2 \geq 0$

$|c_n|^2 = n$

$\boxed{c_n = \sqrt{n}}$

$\hat{a}|0\rangle = 0$

$\sqrt{n}\cdot\sqrt{n+1}$

$\sqrt{n(n+1)} = n$

$n^2 = n(n+1)$

$n = n+1$

c) $\hat{N}\hat{a}|n\rangle = \hat{a}^\dagger\hat{a}\cdot\hat{a}|n\rangle = (\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger)\hat{a}|n\rangle = (\{\hat{a}^\dagger,\hat{a}\} - \hat{a}\hat{a}^\dagger)\hat{a}|n\rangle$

$(1 - \hat{a}\hat{a}^\dagger)\hat{a}|n\rangle = -\hat{a}|n\rangle - \hat{a}\cdot n|n\rangle = \boxed{(-n+1)\hat{a}|n\rangle}$

$\boxed{n' = -n+1}$

$\hat{N}\hat{a}^\dagger|n\rangle = \hat{a}^\dagger\hat{a}\,\hat{a}^\dagger|n\rangle = \hat{a}^\dagger(\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a})|n\rangle$

$= \hat{a}^\dagger(\{\hat{a},\hat{a}^\dagger\} - \hat{a}^\dagger\hat{a})|n\rangle = \hat{a}^\dagger(1 - \hat{a}^\dagger\hat{a})|n\rangle = \hat{a}^\dagger(1-n)|n\rangle = \boxed{(1-n)\hat{a}^\dagger|n\rangle}$

$\boxed{n'' = (1-n)}$

$\hat{a}|n\rangle = c_n|-n-1\rangle$

$n = \langle n|N|n\rangle = \langle n|\hat{a}^\dagger\hat{a}|n\rangle = c_n\langle n|\hat{a}^\dagger|-n-1\rangle$

$= c_n\langle -n-1|\hat{a}|n\rangle^* = |c_n|^2\langle -n-1|-n-1\rangle$

$\boxed{\hat{a}|n\rangle = \sqrt{n}|n'\rangle}$

$c_n = \sqrt{n}$

d) outro valor real

$\sqrt{1-n} \geq 0 \Rightarrow n \leq 1$

$\sqrt{n} \geq 0 \Rightarrow n \geq 0$

$\Rightarrow \boxed{n = 0 \text{ e } n = 1}$

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Q12

a) $u(T) = aT^4$

$U = u(T).V$ e $S = s(T).V$, $P = \frac{u(T)}{3}$

$dU = dQ - dW$

$dU = TdS - pdV$

$\frac{dU}{dV} = T\left(\frac{\partial S}{\partial V}\right)_T - P$

$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$u(T) = T\left(\frac{\partial P}{\partial T}\right)_V - P$

$u(T) = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right)_V - \frac{u(T)}{3} \rightarrow 3u(T) + u(T) = \frac{T}{\cancel{3}} \frac{\partial u}{\partial T}$

$\frac{dT}{T} = \frac{du}{4u(T)}$

$\boxed{u(T) = AT^4}$

b) ii $u = \int_0^\infty \hbar\omega \cdot g(\omega)\, d\omega$

nº fotões por vol. e energia de ...

$u = \hbar\int_0^\infty K \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1} = \frac{\hbar}{(\beta\hbar)^4} \cdot \int_0^\infty K \frac{v^3 dv}{e^v - 1} = \frac{\pi^4 K}{15\hbar^3} \cdot kT^4 = \sigma T^4$

$v = \beta\hbar\omega \quad dv = \beta\hbar$

b) $Z = \sum_n \left(e^{-\beta\varepsilon_q}\right)^n = \frac{1}{1-e^{-\beta\varepsilon_q}} \Rightarrow \ln Z = -\ln\left(1 - e^{-\beta\varepsilon_q}\right)$

$\bar{n}_q = -\frac{1}{\beta} \cdot \left[\frac{1}{1-e^{-\beta\varepsilon_q}} \cdot \left(-e^{-\beta\varepsilon_q} \cdot (-\beta)\right)\right] = +\frac{e^{-\beta\varepsilon_q}}{1-e^{-\beta\varepsilon_q}} = +\frac{1}{e^{\beta\varepsilon_q} - 1}$